PREDICTING RESERVOIR PERFORMANCE USING STOCHASTIC MONTE CARLO SIMULATION

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ABSTRACT

Predicting reservoir performance using conventional deterministic models can be tasking, especially for very complicated reservoir systems. Our paper presents the use of Monte Carlo model for simulations; logic and analysis to achieve useful probabilistic stochastic simulations results in a very efficient and visual manner with Microsoft excel spreadsheet. A Monte Carlo simulation run was carried out for a typical representative reservoir data. The simple Darcy equation was used as the deterministic model while the normal distribution model was employed as the probabilistic density function model. These models were used to construct a stochastic simulation algorithm to predict performance of reservoir systems. Our work analysed 5000 runs using random inputs. By using random inputs, we are essentially turning the deterministic model into a stochastic model which is then solved iteratively over the chosen number of runs. The darcy equation was used as the deterministic model and it was evaluated using a single well data from the reservior data table. A comparison of the deterministic result [5,219.83], for the single run gave a stochastic value of [3,565.34] for the Monte Carlo Simulation [5430.49], and second run gave a more accurate stochastic value [5,234.07], shows that multiple run can achive closeness to actual result through the Monte Carlo Method. The result being quite close to the deterministic value on second run demonstrated that the Monte Carlo iteration can achive a high enough to reliable estimates multiple runs. Various statistical simulators were also employed to display the standard deviation of the range of generated data from the mean and the standard error was also calculated.

* Corresponding author, key words, reservoir performance, stochastic, Monte Carlo, simulations

Introduction

In today's world of oil industries, economic production of oil and gas resources requires carefully engineered recovery projects of increasing technical complexity and sophistication. Hydrocarbons do not reside in cavernous pools awaiting discovery. Rather they are found —sometimes at enormous depths— within the confines of tiny pores in rocks. Although the pores may be interconnected, the resulting pathways still present a significant resistance to the flow of oil toward a well, drilled into the hydrocarbon-bearing strata. In addition, since water resides in some of the pores, hydrocarbons and water are recovered simultaneously at the well-head. Thus, even when large amounts of oil are known to be in a reservoir, often only a relatively small fraction of it can be recovered with conventional pumping technology.

The most common method of enhancing oil recovery, which accounts for much of the oil production in the Nigeria, is the injection of water or gas at strategic locations to displace the oil toward the production wells.

Modern recovery projects of this type require very large capital investments. A single off-shore well drilled to a depth of 15,000 feet can cost up to $100 million. To be successful, a project must have a sizable hydrocarbon target, the promise of extracting oil or gas at a sufficiently high rate, a strategy for water separation and disposal, and a scheme for transportation to a refinery. As exploration efforts are driven to more remote environments, the infrastructure costs for hydrocarbon production escalate. Large potential returns are inevitably accompanied by large monetary risk.

Obviously, economic analysis must be performed before deciding whether a particular investment should be made. Computer simulations of reservoir flow performance are one of the essential ingredients in the analysis. On the basis of both field and laboratory data about the distributions of different rock types and the properties of each type, attempt are made to predict the average, or bulk, flow behavior of the fluids through the hydrocarbon-bearing rock of a reservoir.

By the careful application of the knowledge of reservoir and the behavior of the contained hydrocarbon at the prevalent conditions, the reservoir engineer seeks to manage the reservoir for continuous production of crude oil with a balance between efficient and high yielding technology and the economics of the whole operation with their environmental implications. From the initial prospecting for oil, to the sinking of the wildcat and the developmental wells till the entire life cycle of the oil field is experienced and the stage of oil well plugging and field abandonment is attained, the knowledge of petroleum reservoir management assist the engineer to monitor the wells, decide on production steps and the oil recovery methods to be employed from the primary stage to the tertiary level so as ensure maximum take from the pay.

Thus, much effort has been put into developing software systems that allow the use of the simulation models to predict performance as well as provide the optimization tools and techniques to allow the robust designs to be determined.

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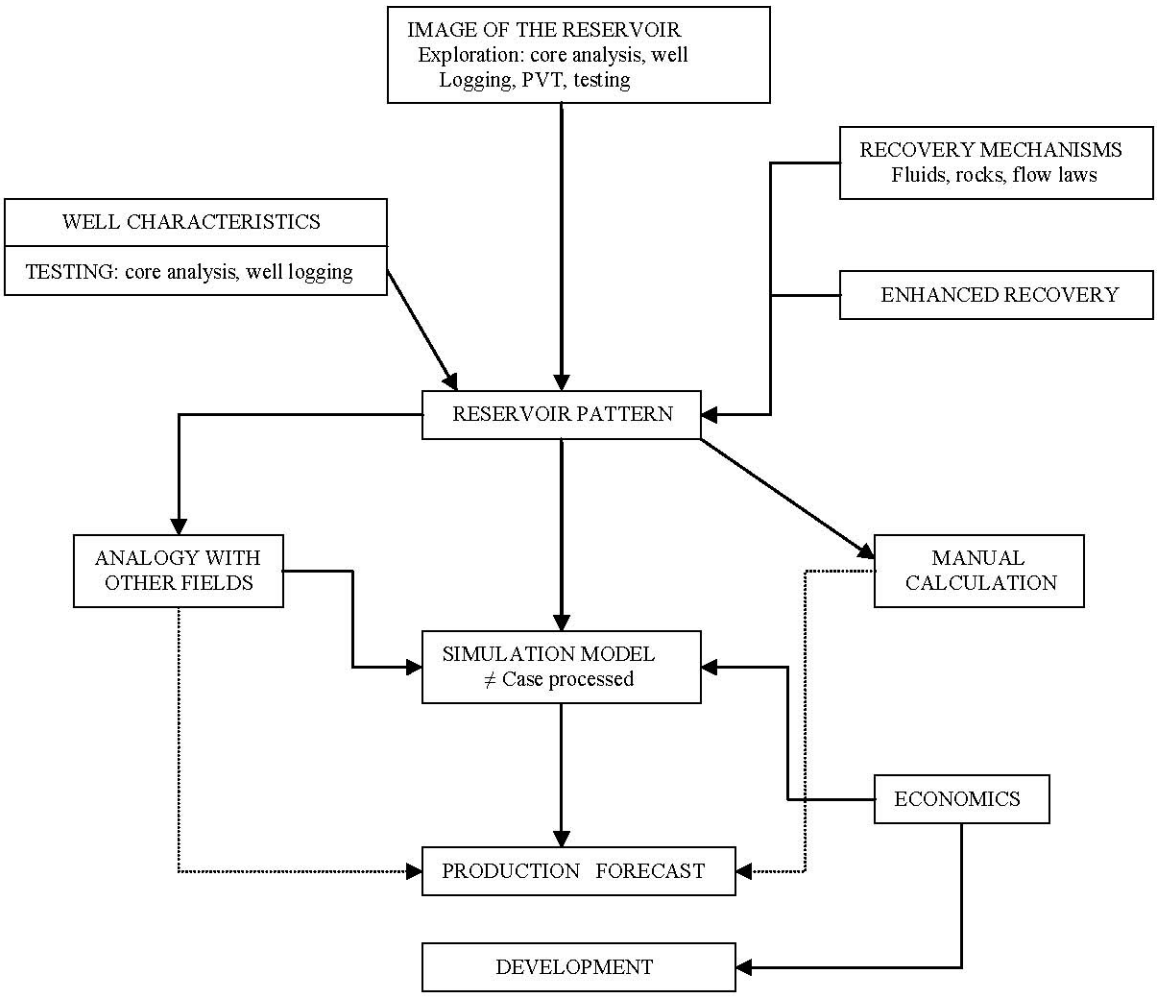

Figure 1: Reservoir Planning Flow Chart

Initially, reservoir engineering was considered the only item of technical importance in managing a hydrocarbon reservoir. Two key points were emphasized --- clear thinking using fundamental reservoir-mechanics concepts and automation using basic computers. The other is the concept of managing oil and gas reservoirs which has evolved more toward the integration of reservoir engineering with other scientific disciplines, namely geology and geophysics (Wyllie, M.R.J., 1962).

The science of reservoir engineering coupled with computer simulation and prediction will continue to evolve; newer and better methods of predicting reservoir behavior will be found. Simply recognizing that integration is beneficial will not be sufficient. True, integration will require persistence (Satter, A., 1992), while a comprehensive program for reservoir management is desirable, every reservoir may not warrant a detailed

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program because it might not be cost-effective. In these cases, reservoir engineering alone or computer simulation may be sufficient.

The purpose of this paper is to provide an overview and application of a simulation technique for reservoir performance prediction. The technique employed a stochastic Monte Carlo simulation using measured reservoir data to simulate an acceptable degree of performance expectation from the analyzed reservoirs. Comparison were also made between the simulated data and actual reservoir values for relevance

Literature Review of Reservoir Models and Simulation Techniques

In recent years, new techniques have appeared in all areas of simulation, including gridding, fluid modeling, numerical approximations, linear solvers, reservoir and geological modeling, etc. All of these technical advances are making simulation more accurate and realistic, and the simulator more robust, fast, stable, and user-friendly.

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Gridding

In the early stages of this technology, all reservoir simulations were performed on rectangular Cartesian grid, radial grid was developed later to simulate near well flow. Local grid refinement was developed to achieve better accuracy in high flow regions (Ciment and Sweet, 1973; Nacul, 1991; Pedrosa and Aziz, 1985). Development of corner-point geometry grid (Ponting, 1989; Ding and Lemonnier, 1995; Peaceman, 1996) made it possible to use non-rectangular gridblocks, providing the capability to model faults and other complex geological features. Up to this point, all grids were structured, where the neighbors of a gridblock could be easily identified from their i, j, k indices. In the last decade, unstructured grids (Heinemann and Brand, 1989; Palagi and Aziz, 1994; Gunasekera et al., 1997; Kocberber, 1997; Aavatsmark et al., 1997; Verma and Aziz, 1996-1997) were introduced, which can easily force gridblocks to conform to major geological features. In unstructured grids, the connections between gridblocks are flexible, and a connection list is used to keep track of the connected gridblocks. Most unstructured grids are non-orthogonal grids, which often require the use of multi-point flux approximations. Today, users can define major reservoir units to which the simulation grid must conform, within each unit, the grid can be generated automatically with some guiding input from the user (Geoquest, 2000). Automatic gridding software packages are becoming available from commercial vendors, such as, FloGrid from Geoquest, GOCAD from the GOCAD group and GridPro from Program Development Corporation (PDC).

In the last several years, flow based unstructured grids (Agut et al., 1998; Edwards et al., 1998; Castellini et al., 2000) have been proposed, which are normally formed by generating streamlines and iso-potential lines from a fine grid single-phase simulation. Considerable smoothing is required to make the grid suitable for simulation. These grids roughly follow major geological features, such as faults, and they are concentrated in high flow rate areas, such as wells or high permeability regions. Edwards et al. (1998) showed that, if the grids follow the streamlines exactly, the multi-point flux calculations can be reduced to two-point flux calculations.

Beside developments in gridding techniques, Lim et al. (1994) proposed a new approach to the representation of grid information. In the conventional approach or the block-based approach, gridblocks and wells are tracked when computing flux terms. While in the new connection based approach, the network of connections are considered. A connection always links two nodes; each node can be a gridblock node, a well node or a surface facility node. A cell (gridblock) list is used for the calculation of accumulation terms, and a connection list is used for the calculation of flux terms. This connection-based approach is extremely convenient for unstructured grids, since the connection list itself has no structure at all. Besides this, it is also suitable for domain decomposition, surface facility modeling (Lim et al., 1994) and multi-point flux calculations.

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2.2 Fluid Modeling

Initially, all simulations were based on the black-oil fluid model, where the hydrocarbon system is represented by two pseudo-components, oil and gas, according to their status at standard conditions. In the early 1980s, compositional simulation, where the hydrocarbon system is represented by an arbitrary number of components and pseudo-components (e.g. Aziz, 1996), became more mature and ready to use. The development of compositional simulation makes it possible to simulate volatile oil reservoirs, CO2 flooding and other EOR processes. However, compositional simulation is much more expensive than black-oil simulation, due to the larger number of unknowns per gridblock and complex flash behavior.

2.3 Numerical Approximations

Conventional simulators use finite-difference methods with two-point flux calculations. More recently, multi-point flux calculations (Verma and Aziz, 1996; Gunasekera et al., 1998) are becoming more and more common due to the use of full tensor permeability (Lee et al., 1994; Lee et al., 1997) and non-orthogonal grids. At the same time, control volume methods (Aavatsmark et al., 1997; Verma and Aziz, 1997) have become the method of choice for today's simulator, because of their easy handling of unstructured grids. Limited work has also been done on higher-order schemes (Sammon, 1991; Chen et al., 1991) and finite element methods (Young, 1978; Fung et al., 1991; Sukirman and Lewis, 1994) to achieve higher accuracy.

2.4 Linear Solver

Solving the linear system is the single most costly part for a simulation. So it is extremely important to have a good linear solver. Initially, direct solvers were used, but as the problems have become larger and larger (more and more gridblocks), iterative solvers have become more and more common. The performance of iterative solvers depends on the quality of preconditioners. In the petroleum industry, most of the effort in this area has been on building better preconditioners. Any iterative solver can be used as a preconditioner for other iterative solvers, and different preconditioners can be combined together to form multistage preconditioners. Traditional preconditioners include, Incomplete LU decomposition (ILU) (e.g. Behie and Forsyth, 1983), Gauss-Sediel (GS) (e.g. Aziz and Settari, 1979), Algebraic Multi-Grid (AMG) (Stueben, 1983), etc. Their performance depends strongly on the nature of the linear system. For near-elliptic system, AMG works well, for near-hyperbolic system; both ILU and GS work well. In reservoir simulation, we have a near-elliptic pressure equation and near-hyperbolic saturation equations. For this kind of mixed system, none of the single stage preconditioners (ILU, GS, AMG, etc) work well, we need a smarter preconditioner. Wallis et al. (1985) introduced the Constrained Pressure Residual (CPR) preconditioner, which is specially designed for reservoir simulation equations. It uses a two-stage approach to solve the pressure part and the saturation part of the reservoir equations separately, which make it the most promising preconditioner for fully implicit simulations. Unfortunately, a good preconditioner is still missing for the Adaptive Implicit (AIM) Method (Forsyth and Sammon, 1986).

Most of the available commercial linear solvers have been designed for structured grids, which result in Jacobian matrices with a banded structure. For unstructured grids, the Jacobian matrix is a general sparse matrix, and for such systems the existing solvers loose their efficiency.

2.5 Reservoir and Geological Modeling

Reservoir simulations have also expanded in terms of options and features. Modern reservoir simulators can simultaneously handle multiple reservoirs, surface facilities (Schiozer and Aziz, 1994; Byer, 2000) and rock mechanics. Furthermore, there is more and more cooperation among reservoir engineer, geologist and geophysicist to achieve more realistic modeling of reservoir geology for simulation projects (e.g. Journel, 1990; Ballin et al., 1993).

With all of these developments in simulation technology and computers, the simulator has also reached a new stage. Today's simulator is more robust, more complex, and easier to use than what was available 20 years ago. Because of all the features that are required in a modern simulator, it takes more time to develop a simulator and more effort to maintain it. In the future, simulators will be required to solve even more complex problems than they can handle today. Under these circumstances, a good design for the simulator is vital. Fortunately, today we have all kinds of design tools and suitable computer languages to develop technology to meet the future needs of the industry. Research students working in this area need a lot of time to program the basic simulator before they can even start to explore research topics of interest to them. Also, after a student leaves most of his/her development becomes unusable within a short time, due to lack of good design and documentation. Hence developing a good environment for simulation research has become essential for a research group.

STATISTICAL SIMULATION TECHNIQUE: MONTE CARLO

Numerical methods that are known as Monte Carlo methods can be loosely described as statistical simulation methods, where statistical simulation is defined in quite general terms to be any method that utilizes sequences of random numbers to perform the simulation. Monte Carlo is now used routinely in many diverse fields, from the simulation of complex physical phenomena such as radiation transport in the earth's atmosphere and the simulation of the esoteric sub-nuclear processes in high energy physics experiments.

PROBABILITY DENSITY FUNCTIONS [PDFs]

Statistical simulation methods may be contrasted to conventional numerical discretization methods applied to ordinary or partial differential equations that describe some underlying physical or mathematical system. In many applications of Monte Carlo, the physical process is simulated directly, and there is no need to even write down the differential equations that describe the behavior of the system. The only requirement is that the physical (or mathematical) system be described by probability density functions (pdf's). For now, we will assume that the behavior of a system can be described by pdf's. Once the pdf's are known, the Monte Carlo simulation can proceed by random sampling from the pdf's. Many simulations are then performed (multiple ``trials'' or ``histories'') and the desired result is taken as an average over the number of observations (which may be a single observation or perhaps millions of observations). In many practical applications, one can predict the statistical error (the ``variance'') in this average result, and hence an estimate of the number of Monte Carlo trials that are needed to achieve a given error.

Assuming that the evolution of the physical system can be described by probability density functions (pdf's), then the Monte Carlo simulation can proceed by sampling from these pdf's, which necessitates a fast and effective way to generate random numbers uniformly distributed on the interval [0,1]. The outcomes of these random samplings, or trials, must be accumulated or tallied in an appropriate manner to produce the desired result, but the essential characteristic of Monte Carlo is the use of random sampling techniques (and perhaps other algebra to manipulate the outcomes) to arrive at a solution of the physical problem. In contrast, a conventional numerical solution approach would start with the mathematical model of the physical system, discretizing the differential equations and then solving a set of algebraic equations for the unknown state of the system.

It should be kept in mind though that this general description of Monte Carlo methods may not directly apply to some applications. It is natural to think that Monte Carlo methods are used to simulate random, or stochastic processes, since these can be described by pdf's. However, this coupling is actually too restrictive because many

Monte Carlo applications have no apparent stochastic content, such as the evaluation of a definite integral or the inversion of a system of linear equations. However, in these cases and others, one can present the desired solution in terms of pdf's, and while this transformation may seem artificial, this step allows the system to be *treated* as a stochastic process for the purpose of simulation and hence Monte Carlo methods can be applied to simulate the system. Therefore, we take a broad view of the definition of Monte Carlo methods and include in the Monte Carlo rubric all methods that involve statistical simulation of some underlying system, whether or not the system represents a real physical process.

Major Components of a Monte Carlo Algorithm

The primary components of a Monte Carlo simulation method include the following:

- *Probability distribution functions (pdf's)* - the physical (or mathematical) system must be described by a set of pdf's.
- *Random number generator* - a source of random numbers uniformly distributed on the unit interval must be available.
- *Sampling rule* - a prescription for sampling from the specified pdf's, assuming the availability of random numbers on the unit interval, must be given.
- *Scoring (or tallying)* - the outcomes must be accumulated into overall tallies or scores for the quantities of interest.
- *Error estimation* - an estimate of the statistical error (variance) as a function of the number of trials and other quantities must be determined.
- *Variance reduction techniques* - methods for reducing the variance in the estimated solution to reduce the computational time for Monte Carlo simulation
- *Parallelization and vectorization* - algorithms to allow Monte Carlo methods to be implemented efficiently on advanced computer architectures.

The Monte Carlo method is just one of many methods for analyzing uncertainty propagation, where the goal is to determine how different factors affect the *sensitivity, performance,* or *reliability* of the system that is being modeled. Monte Carlo simulation is categorized as a sampling method because the inputs are randomly generated from *probability distributions* to simulate the process of sampling from an actual *population.* So, we try to choose a distribution for the inputs that most closely *matches data we already have,* or best represents our *current state of knowledge*. The data generated from the simulation can be represented as probability distributions (or histograms) or converted to *error bars*, *reliability predictions*, *tolerance zones*, and *confidence intervals*.

The steps in Monte Carlo simulation corresponding to the uncertainty propagation are fairly simple, and can be easily implemented in Excel for simple models. All we need to do is follow the chart below:

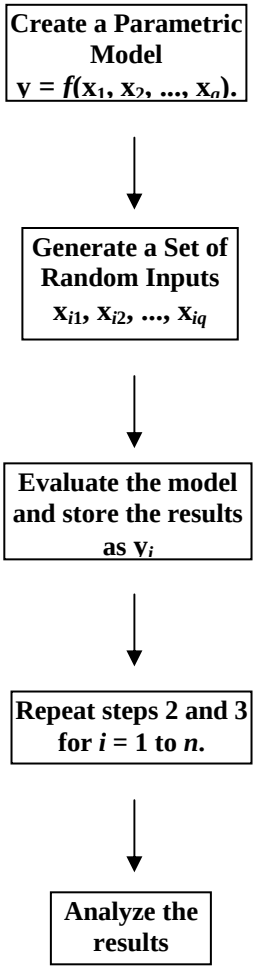

Figure 2: Steps in Monte Carlo methods

MONTE CARLO MODEL SIMULATION METHOD OF TYPICAL RESERVOIR

In achieving the Monte Carlo simulation, an actual reservoir database containing information about various oil wells in an oil field was obtained and processed. A sample of the database was extracted for comparison with the Monte Carlo simulation as shown.

Table 1.0 Representative reservoir data employed for Monte Carlo simulations

REPRESENTATIVE RESERVOIR DATA EMPLOYED FOR MONTE CARLO SIMULATIONS

SN	FIELD NAME	GRAVITY	VISCOUSITY	TEMPERATURE	POROSITY	PERMEABILITY	NET PAY	ANN. PROD.
		API	cp	T	%	md	Ft	bbl
1	Careyville Landing	19	29	135	27	333	7	8,315
2	Lewisville, West	19	40	125	20	650	14	1,671
3	Mayton	16	43	139	26	540	12	7,838
4	Midway, East	18	79	113	25	940	6	6,417
5	Pigeon Hill	15	36	151	23	330	13	3,249
6	Winchester	20	65	121	28	709	10	3,393
7	Edison, Main Area	18	97	109	26	675	25	2,333
8	Sansinena, New England Area	18	28	142	33	200	150	429
9	Tejon, Western Area	19	28	135	28	510	60	2,706
10	Bayou Middle Fork	16	23	163	15	250	4	4,549
11	Avera	20	7	198	20	560	16	8,217
12	Cedro Hill	19	30	101	31	700	12	7,215
13	Chapel Hill, East	16	14	182	23	280	13	4,864
14	Forest Hill	16	23	162	24	200	8	2,015
15	Manziel	18	21	153	24	200	13	4,340
16	McCrary, West	17	22	157	24	200	7	6,779
17	Nolan Edward	20	12	165	24	250	26	2,126
18	Norman Paul	18	15	174	25	500	22	4,645
19	Nova	20	12	165	24	200	15	3,417
20	Nova, South	19	14	164	24	113	10	1,644
21	Quitman, South	16	28	154	24	200	7	4,961
22	Raccoon Bend	19	39	126	25	500	15	6,554
23	Sugar Hill	18	16	166	22	375	11	3,804
24	Tully	17	21	159	24	300	15	2,132

* Corresponding author, key words, reservoir performance, stochastic, Monte Carlo, simulations

For the purpose of this simulation a model equation is required for the application of the Monte Carlo method. The equation employed is the Darcy equation for the prediction of flow rate in a porous medium using the medium's permeability, viscosity, area, length of section and the pressure drop as the required input. The equation is shown below.

Darcy equation is given as**:**

$$q = \frac{KA}{\mu} \times \frac{\Delta P}{L} \qquad 1$$

q = Production Rate
k = Permeability
A = Cross section area
μ = Fluid viscosity
$\frac{\Delta P}{L}$ = Pressure gradient (Pressure change per unit length).

The simulation was carried out using Microsoft office excel version 2003. This is a spread sheet application used for mathematical and statistical analysis. Its ability to calculate a large range of data and its relevant in-built functions was employed. The equation was then rearranged to calculate the Change in pressure using the MS Excel cell reference formula as shown in table below.

Table 2.0 Calculation of pressure change using Darcy law

CALCULATION OF PRESSURE CHANGE USING DARCY LAW

SN	FIELD NAME	GRAVITY	VISCOUSITY	TEMPERATURE	POROSITY	PERMEABILITY	NET PAY	ANN. PROD.	**PRESSURE CHANGE**
		API	Cp	T	%	md	ft	Bbl	**psi**
1	Careyville Landing	19	29	135	27	333	7	8,315	**5069**
2	Lewisville, West	19	40	125	20	650	14	1,671	**1440**
3	Mayton	16	43	139	26	540	12	7,838	**7490**
4	Midway, East	18	79	113	25	940	6	6,417	**3236**
5	Pigeon Hill	15	36	151	23	330	13	3,249	**4608**
6	Winchester	20	65	121	28	709	10	3,393	**3111**
7	Edison, Main Area	18	97	109	26	675	25	2,333	**8382**
8	Sansinena, New England Area	18	28	142	33	200	150	429	**9009**
9	Tejon, Western Area	19	28	135	28	510	60	2,706	**8914**
10	Bayou Middle Fork	16	23	163	15	250	4	4,549	**1674**
11	Avera	20	7	198	20	560	16	8,217	**1643**
12	Cedro Hill	19	30	101	31	700	12	7,215	**3711**
13	Chapel Hill, East	16	14	182	23	280	13	4,864	**3162**
14	Forest Hill	16	23	162	24	200	8	2,015	**1854**
15	Manziel	18	21	153	24	200	13	4,340	**5924**
16	McCrary, West	17	22	157	24	200	7	6,779	**5220**
17	Nolan Edward	20	12	165	24	250	26	2,126	**2653**
18	Norman Paul	18	15	174	25	500	22	4,645	**3066**
19	Nova	20	12	165	24	200	15	3,417	**3075**
20	Nova, South	19	14	164	24	113	10	1,644	**2037**
21	Quitman, South	16	28	154	24	200	7	4,961	**4862**
22	Raccoon Bend	19	39	126	25	500	15	6,554	**7668**
23	Sugar Hill	18	16	166	22	375	11	3,804	**1785**
24	Tully	17	21	159	24	300	15	2,132	**2239**

The pressure changes were then normalized i.e. represented with a normal distribution and the corresponding probability was calculated. This calculation also achieved with MS Excel functions required the calculation of the mean and standard deviation of the set of pressure changes for the different reservoirs.

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Table 3.0 Presentation of pressure change as a probability density function

PRESENTATION OF PRESSURE CHANGE AS A	MEAN [∇P]:	4242.88
PROBABILITY DENSITY FUNCTION	STANDARD DEVIATION [∇P]:	2465.29

SN	FIELD NAME	VISCOUSITY	PERMEABILITY	NET PAY	Q ANNUAL. PROD.	∇P PRES. CHANGE	NORMAL DISTRIBUTION	PROBABILITY
		cp	md	ft	bbl	psi	z of ∇P	
1	Careyville Landing	29	333	7	8,315	5069	0.3351	0.6312
2	Lewisville, West	40	650	14	1,671	1440	-1.1371	0.1278
3	Mayton	43	540	12	7,838	7490	1.3170	0.9061
4	Midway, East	79	940	6	6,417	3236	-0.4085	0.3415
5	Pigeon Hill	36	330	13	3,249	4608	0.1480	0.5588
6	Winchester	65	709	10	3,393	3111	-0.4593	0.3230
7	Edison, Main Area	97	675	25	2,333	8382	1.6788	0.9534
8	Sansinena, New England Area	28	200	150	429	9009	1.9333	0.9734
9	Tejon, Western Area	28	510	60	2,706	8914	1.8947	0.9709
10	Bayou Middle Fork	23	250	4	4,549	1674	-1.0420	0.1487
11	Avera	7	560	16	8,217	1643	-1.0544	0.1458
12	Cedro Hill	30	700	12	7,215	3711	-0.2159	0.4145
13	Chapel Hill, East	14	280	13	4,864	3162	-0.4386	0.3305
14	Forest Hill	23	200	8	2,015	1854	-0.9691	0.1663
15	Manziel	21	200	13	4,340	5924	0.6820	0.7524
16	McCrary, West	22	200	7	6,779	5220	0.3963	0.6541
17	Nolan Edward	12	250	26	2,126	2653	-0.6448	0.2595
18	Norman Paul	15	500	22	4,645	3066	-0.4775	0.3165
19	Nova	12	200	15	3,417	3075	-0.4736	0.3179
20	Nova, South	14	113	10	1,644	2037	-0.8948	0.1854
21	Quitman, South	28	200	7	4,961	4862	0.2510	0.5991
22	Raccoon Bend	39	500	15	6,554	7668	1.3894	0.9176
23	Sugar Hill	16	375	11	3,804	1785	-0.9969	0.1594
24	Tully	21	300	15	2,132	2239	-0.8130	0.2081

In representing the above data using simulation, random numbers were generated to replicate the probability calculated for each of the above pressure change. Since the probabilities being for a normal distribution are between 0 and 1, random numbers were generated to lie between 0 and 1 also. MS Excel functions were then written to

achieve an inversion of the simulated probability values to pressure changes. To ensure that the simulated values keep dimensions with the actual reservoir data, the mean and standard deviation calculated for the actual data were employed for the inversion.

Table 4 Simulation of probability distribution of pressure changes i.e. Monte Carlo Simulations using mean and standard deviation of actual reservoir data

SIMULATION OF PROBABILITY DISTRIBUTION OF PRESSURE CHANGES i.e. MONTE CARLO SIMULATIONS USING MEAN AND STANDARD DEVIATION OF SAMPLE RESERVOIR DATA

MEAN [∇P]:	4242.88
STANDARD DEVIATION [∇P]:	2465.29

SN	ANNUAL PROD. Q	PRES. CHANGE ∇P	SIMULATED PROBABILITY	SIMULATED NORMAL DIST.	SIMULATED PRES. CHANGE [∇P]	SIMULATED ANNUAL PROD. Q
	bbl	Psi		z of sP	psi	bbl
1	8,315	5069	0.6687	0.4362	5318.23	8723.99
2	1,671	1440	0.2741	-0.6006	2762.28	3206.22
3	7,838	7490	0.0056	-2.5333	-2002.57	-2095.71
4	6,417	3236	0.8560	1.0624	6861.99	13608.16
5	3,249	4608	0.2947	-0.5398	2912.22	2053.49
6	3,393	3111	0.9244	1.4353	7781.41	8487.72
7	2,333	8382	0.6607	0.4143	5264.13	1465.27
8	429	9009	0.4698	-0.0757	4056.15	193.15
9	2,706	8914	0.6686	0.4362	5318.13	1614.43
10	4,549	1674	0.2936	-0.5429	2904.46	7892.55
11	8,217	1643	0.5796	0.2009	4738.13	23690.66
12	7,215	3711	0.8913	1.2336	7283.99	14163.31
13	4,864	3162	0.0309	-1.8681	-362.52	-557.72
14	2,015	1854	0.8004	0.8431	6321.34	6871.02
15	4,340	5924	0.3393	-0.4142	3221.65	2360.18
16	6,779	5220	0.2909	-0.5508	2884.87	3746.59
17	2,126	2653	0.1423	-1.0702	1604.58	1285.72
18	4,645	3066	0.3568	-0.3670	3338.21	5057.89
19	3,417	3075	0.9973	2.7780	11091.39	12323.77
20	1,644	2037	0.7030	0.5332	5557.31	4485.54
21	4,961	4862	0.1203	-1.1733	1350.37	1377.93
22	6,554	7668	0.1139	-1.2060	1269.66	1085.18
23	3,804	1785	0.2184	-0.7777	2325.58	4955.06
24	2,132	2239	0.2730	-0.6038	2754.42	2623.26

From table above, the calculated change was then employed in the Darcy equation to find the simulated production rate as shown in same table. Table 5 below shows a comparison of the reservoir data and its derivation to the simulated values and the calculations made with these values.

* Corresponding author, key words, reservoir performance, stochastic, Monte Carlo, simulations

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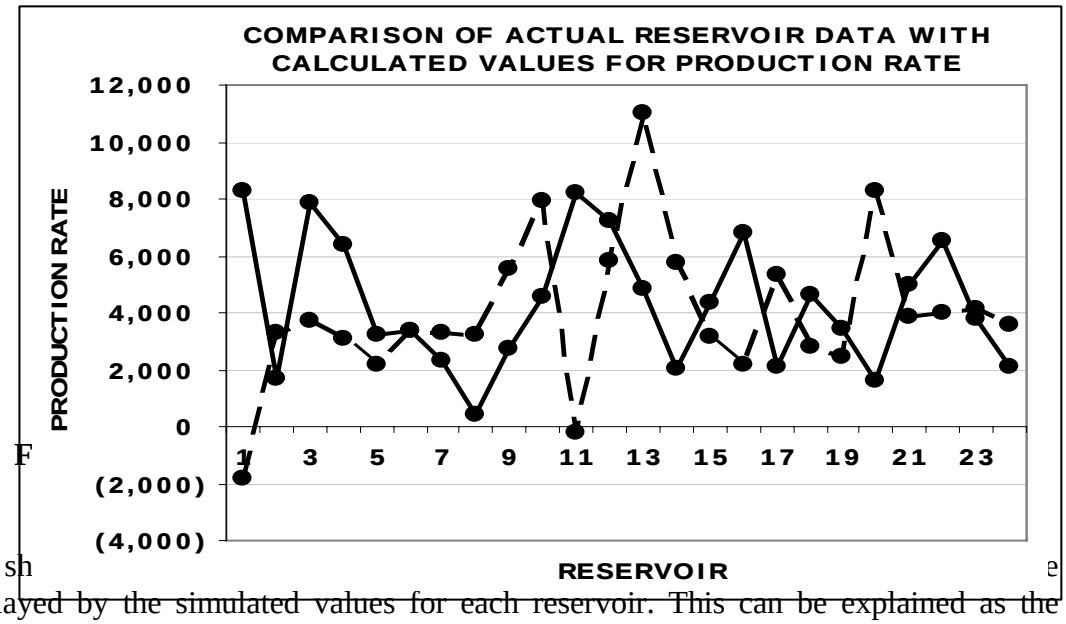
Table 5 Comparison of actual and simulated values of production rate and pressure change

COMPARISON OF ACTUAL AND SIMULATED VALUES OF PRODUCTION RATE AND PRESSURE CHANGE

		SIMULATED	SIMULATED
ANNUAL PROD. Q	PRES. CHANGE ▽P	ANNUAL PROD. Q	PRES. CHANGE ▽P]
bbl	Psi	Bbl	psi
8,315	5069	8498.70	13941.21
1,671	1440	3516.81	4082.02
7,838	7490	3878.28	4058.66
6,417	3236	7037.80	13956.83
3,249	4608	2960.34	2087.42
3,393	3111	4489.42	4896.92
2,333	8382	4453.27	1239.57
429	9009	6926.12	329.82
2,706	8914	6363.72	1931.84
4,549	1674	6832.24	18565.86
8,217	1643	8613.96	43069.79
7,215	3711	4501.12	8752.17
4,864	3162	4084.04	6283.13
2,015	1854	4748.70	5161.63
4,340	5924	2968.70	2174.87
6,779	5220	5730.38	7442.05
2,126	2653	4970.26	3982.58
4,645	3066	6157.29	9329.22
3,417	3075	4438.73	4931.92
1,644	2037	3193.34	2577.48
4,961	4862	2058.07	2100.07
6,554	7668	1049.52	897.03
3,804	1785	1721.26	3667.46
2,132	2239	1548.43	1474.69

The similarities between the two values are clearly observed despite the slight variation. It should be noted that series of runs of the MS Excel calculations result in successive evaluation of the formula using the random numbers generated. This can be better observed when the program is run on MS Excel (with continuous hitting of F9 key), the values is seen to change as the whole worksheet is recalculated with new random numbers each time. The plot of the two quantities produced and pressure change values are shown below. The graph when run on the MS Excel worksheet is also recalculated each time.

* Corresponding author, key words, reservoir performance, stochastic, Monte Carlo, simulations



F

The sh e displayed by the simulated values for each reservoir. This can be explained as the outcome of very low pressure changes in the well. Since this is the result from a simulated case as we can observe that data for the actual reservoir are above the negative lines (the thick continuous lines), though often, the simulated result results in all positive production (achieved by hitting the F9 key). In actual reservoirs, low pressures results in no production. This is the main thrust of the secondary and tertiary recovery techniques. This can therefore be used to compare reality and simulation.

A further graph is also plotted to show pressure variation between the reservoir data and the simulation. The pattern is seen to also represent an average of the actual reservoir data as shown in Figure 4

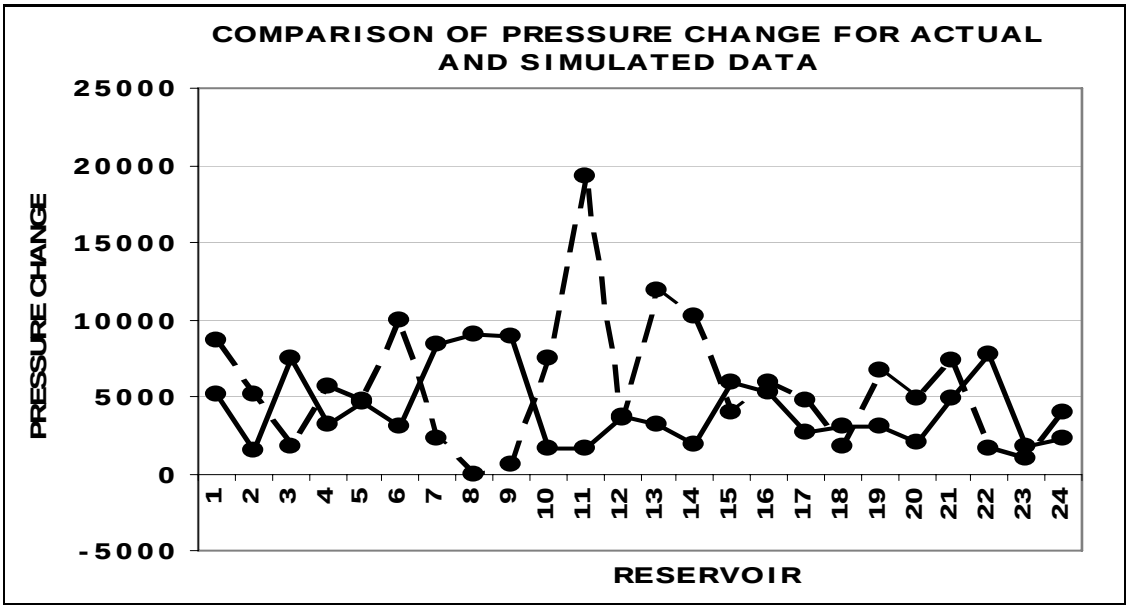


Figure 4 Comparison of actual reservoir data with simulated values for pressure change

The following graphs show the statistics of the number of reservoir with particular classes of production and pressure for a single run. It is obvious that majority of the reservoirs have production rate production rate above average for most of the time as seen from the frequency. The graph of pressure is also similar.

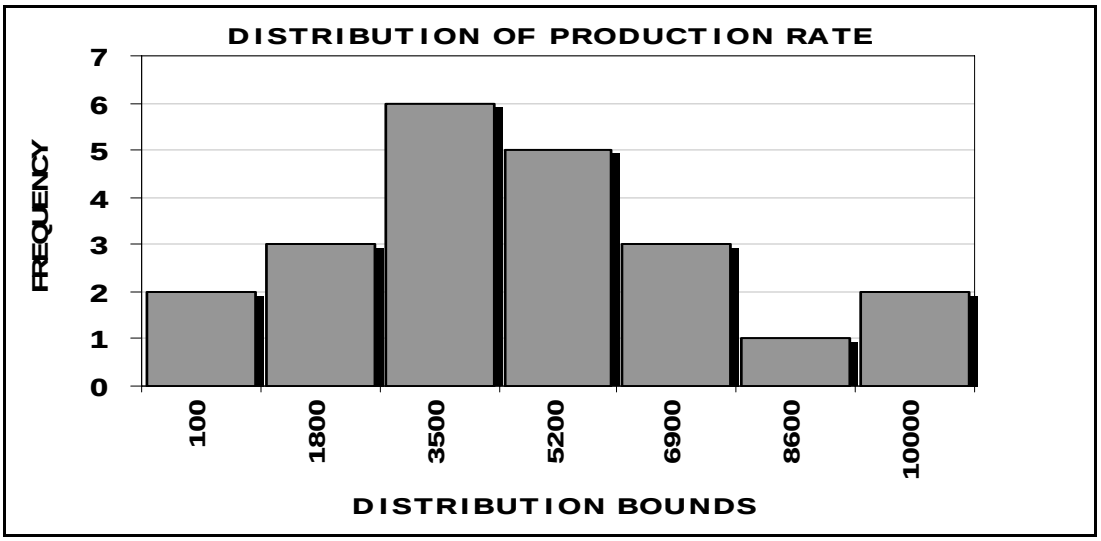


Figure 5 Distribution of production rate

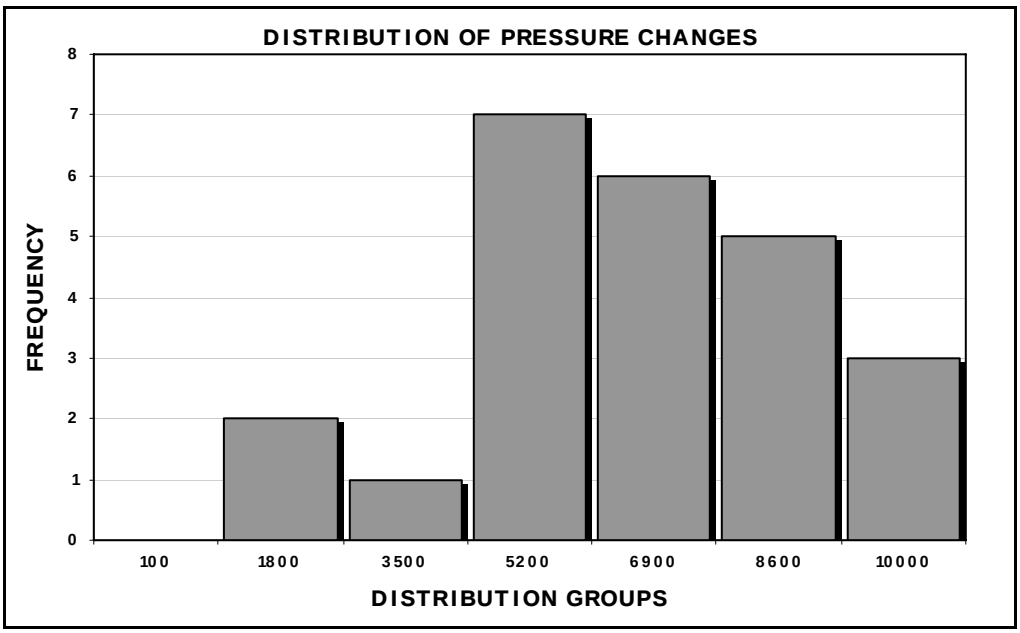


Figure 6 Distributions of pressure changes.

Monte Carlo Stochastic Simulation: Results and Discussions

The main ingredient of a stochastic simulation using Monte Carlo is the analysis of a particular problem using randomly generated data. When a model created with a spreadsheet like MS Excel, we have a certain number of *input parameters* and a few equations that use those inputs to give a set of *outputs* (or *response* variables). This type of model is usually **deterministic**, meaning that we get the same results no matter how many times you re-calculate as shown below.

Monte Carlo simulation is a method for *iteratively* evaluating a deterministic model using sets of random numbers as inputs. This method is often used when the model is complex, nonlinear, or involves more than just a couple uncertain parameters. A simulation can typically involve *over 10,000 evaluations* of the model, the higher the number of evaluation, the more accurate the results. This work analyzed 5000 runs using random inputs. By using **random inputs**, we are essentially turning the deterministic model into a stochastic model which is then solved iteratively over the chosen number of runs.

The deterministic model used is the Darcy equation and it was evaluated using a single well data from the sample shown earlier.

SN	FIELD NAME	GRAVITY	VISCOUSITY	TEMPERATURE	POROSITY	PERMEABILITY	NET PAY	ANN. PROD.
		API	Cp	T	%	md	ft	bbl
16	McCrary, West	17	22	157	24	200	7	6,779

Table 6 Input data for the model

Using the Darcy equation, the pressure change was evaluated. A single run stochastic evaluation was also done using generated data so as to check the correlation.

Table 7 result of model evaluation; deterministic and single run stochastic

Input Values (input)

	Reservoir	Min	Max	Stochastic
Annual Prod. (Q)	6,779	4067.4	8134.8	5484.97
Viscosity (u)	22.00	17.6	28.6	22.69
Net Pay (l)	7.00	5.6	9.8	6.21
Permeability (k)	200.00	180	220	216.63
Unity Area (A)	1.00			

Results (calculations)

	Deterministic	Stochastic
Pressure Change:	5,219.83	3,565.34

Actual reservoir result deterministic

Press F9 to Re-Calculate single run stochastic

Second Run-Press F9-Close to Determined Values

Reservoir Performance Prediction Model

Darcy's Law

Input Values (input)

	Reservoir	Min	Max	Stochastic
Annual Prod. (Q)	6,779	4067.4	8134.8	5174.09
Viscosity (u)	22.00	17.6	28.6	24.24
Net Pay (l)	7.00	5.6	9.8	8.10
Permeability (k)	200.00	180	220	194.03
Unity Area (A)	1.00			

Actual reservoir result deterministic

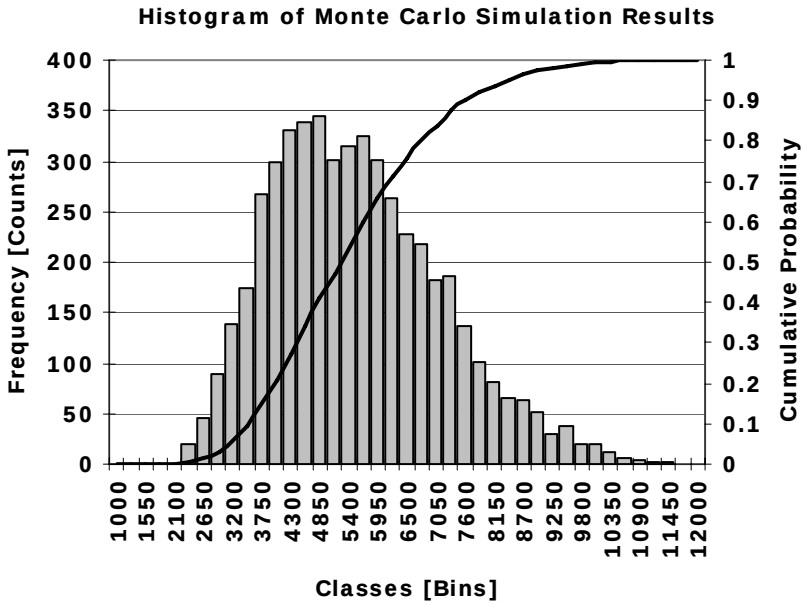
Press F9 to Re-Calculate single run stochastic

Results (calculations)		
	Deterministic	Stochastic
Pressure Change:	5,219.83	5,234.07

The stochastic column was designed to pick random values between the minimum and maximum of each of the parameter. It is recalculated when the program is run on MS Excel.

The actual Monte Carlo simulation was then run for n = 5000, and the model evaluated for each run. A mean of the 5000 pressure changes was evaluated and compared to the result from the deterministic evaluation and the single run stochastic method as shown below.

Monte Carlo Simulation (Summary Report)							See Monte Workshee
Summary Statistics			**Probability of Pressure greater or Less Than**				
		Press F9 to Re-Calculate Monte Carlo simulation				**Actual Reservoir Pressure:**	5,219.83
Sample Size (***n***):	10000					**Simulated Mean:**	5423.92
MEAN:	5,423.92		**Pr(x <**	5219.83	**) =**		49.09%
STDEV:	1,602.44		**Pr(x >**	5219.83	**) =**		50.91%
Mean Standard Error:	22.66						



* Corresponding author, key words, reservoir performance, stochastic, Monte Carlo, simulations

Table 7 Monte Carlo simulation

	RAND	RAND	RAND	RAND	CONSTANT		RESULT
SN	ANN. PROD.	VISCOUSITY	NET PAY	PERMEABILITY	AREA		PRES. CHANGE
1	5919.82	18.9019	7.5285	211.787	1.0000		3977.63
2	4288.08	24.1184	5.9664	199.609	1.0000		3091.32
3	8114.23	23.1584	5.6013	201.906	1.0000		5213.11
4	5699.41	21.9730	9.5099	188.168	1.0000		6329.22
5	7645.83	22.5242	8.2960	204.178	1.0000		6997.32
6	7818.79	19.9602	7.2921	200.208	1.0000		5684.28
7	6006.17	21.7557	6.7421	212.968	1.0000		4136.67
8	4180.42	24.5965	7.0963	208.429	1.0000		3500.77
9	4706.93	22.4741	7.9709	211.560	1.0000		3985.61
10	6649.93	21.3102	6.6919	186.110	1.0000		5095.45
11	6358.01	26.0448	6.2409	188.093	1.0000		5494.35
12	5875.52	28.0539	7.8074	215.126	1.0000		5982.06
13	5815.93	18.1239	7.9285	203.376	1.0000		4109.22
14	5663.84	25.2883	9.4355	183.497	1.0000		7364.91
15	5063.47	22.6199	7.0193	204.029	1.0000		3940.43
16	7441.89	25.9181	9.5390	219.894	1.0000		8367.14
17	7694.57	24.4137	6.0668	207.418	1.0000		5494.59
18	4370.74	20.2703	9.7044	197.716	1.0000		4348.52
19	5115.90	20.0580	7.4200	183.149	1.0000		4157.29
20	8050.68	26.0901	6.2585	200.873	1.0000		6544.21
21	7528.64	19.4813	9.2545	204.023	1.0000		6652.83
22	6814.84	22.8230	5.7473	191.948	1.0000		4656.99
23	4226.86	21.8885	7.2867	184.894	1.0000		3646.22
-	-	-	-	-	-		-
-	-	-	-	-	-		-
-	-	-	-	-	-		-
-	-	-	-	-	-		-
4994	7648.14	22.9760	9.4753	202.159	1.0000		8236.26
4995	5022.32	19.0569	9.2475	195.180	1.0000		4534.65
4996	4624.11	18.4947	5.6660	216.567	1.0000		2237.50
4997	7945.35	28.2878	5.6082	215.965	1.0000		5836.52
4998	7806.47	28.1398	8.8307	215.027	1.0000		9021.53
4999	7816.65	25.0269	6.3036	195.787	1.0000		6298.37
5000	5607.49	19.6907	6.6374	201.791	1.0000		3631.83

Table 8 calculated mean of the Monte Carlo simulation

Summary Statistics	
Sample Size (N):	5000

Central Tendency (Location)			
Mean:	5430.49	Median:	5249.74
StErr:	22.41		

Refer to appendix F for formula used

Although we still need to analyze the data, we have essentially completed a Monte Carlo simulation. Because we have used the volatile RAND () formula, to re-run the simulation all we have to do is recalculate the worksheet each time (F9 is the shortcut).

This may seem like a strange way to implement Monte Carlo simulation, but what is going on behind the scenes is that every time the Worksheet recalculates:

(1) 5000 sets of random inputs are generated

(2) The model is evaluated for all 5000 sets.

(3) The mean of the evaluation is calculated

MS Excel is handling all of the iteration.

A comparison of the deterministic result [5,219.83], the single run stochastic method [3,565.34] and the Monte Carlo simulation [5430.49] shows the advantage of the multiple run achieved through the Monte Carlo method.

Some statistical tools were employed to analyze the result as explained below.

Sample Size (*n*)

The sample size, *n*, is the number of observations or data points from a single MC simulation. For this example, we obtained $n = 5000$ simulated observations. Because the Monte Carlo method is stochastic, if we repeat the simulation, we will end up calculating a different set of summary statistics. The larger the sample size, the smaller the difference will be between the repeated simulations.

Central Tendency: Mean and Median

The sample mean and median statistics describe the central tendency or "location" of the distribution. The arithmetic mean is simply the average value of the observations.

If you sort the results from lowest to highest, the median is the "middle" value or the 50th Percentile, meaning that 50% of the results from the simulation are less than the median. If there is an even number of data points, then the median is the average of the middle two points.

Spread: Standard Deviation, Range, Quartiles

The standard deviation and range describe the spread of the data or observations. The standard deviation is calculated using the STDEV function in MS Excel.

The sample mean is just an estimate of the true population mean. It is obvious that by repeating the simulation (using F9) that the mean is not the same for each simulation.

Standard Error

When the Monte Carlo simulation repeated and the sample means each time recorded, the standard error is a good estimate of the standard deviation of this distribution, assuming that the sample is sufficiently large.

The standard error is calculated using the following formula:

$$StErr = \frac{s}{\sqrt{n}}$$

Range

The range is also a helpful statistic, and it is simply the maximum value minus the minimum value. Extreme values have a large effect on the range, so a better measure of spread is a measure called the Interquartile Range.

The Interquartile Range represents the central 50% of the data. If you sorted the data from lowest to highest, and divided the data points into 4 sets, you would have 4 Quartiles.
The above enumerated tools were then used to analyze the data and comparison was made with the other models i.e. the deterministic model and the single run stochastic model. The summary of the result are shown below.
Table 9 Summary statistics for Monte Carlo simulation

Summary Statistics			
Sample Size (N):	5000		
Central Tendency (Location)			
Mean:	5452.18	Median:	5263.81
StErr:	22.70		
Spread			
StDev:	1605.39		
Max:	11429.06	Q(.75):	6452.98
Min:	2088.49	Q(.25):	4246.42
Range:	9340.57	IQ Range:	2206.55

Refer to appendix G for formula used

As a way of predicting the probability of a particular range of pressure change, the following was calculated.

Table 10 Calculating probabilities using the Excel percent rank function.

Probability of Pressure greater or Less Than			
		Actual Reservoir Pressure:	5,219.83
		Simulated Mean:	5452.18
Pr(x <	5219.83) =		48.75%
Pr(x >	5219.83) =		51.25%

Refer to appendix H for formula used

The accuracy of the result will depend upon the number of data points and how far out on the tails of the distribution is (also on how realistic the model is, how well the input

distributions represent the true uncertainty or variation, and how good the random number generator is). Recalculating the spreadsheet a few times by pressing F9 will give an idea of how much the result may vary between each simulation.

A graph is shown below to give a visual representation of how the data varies and the probability of each group is displayed on the graph for each group.

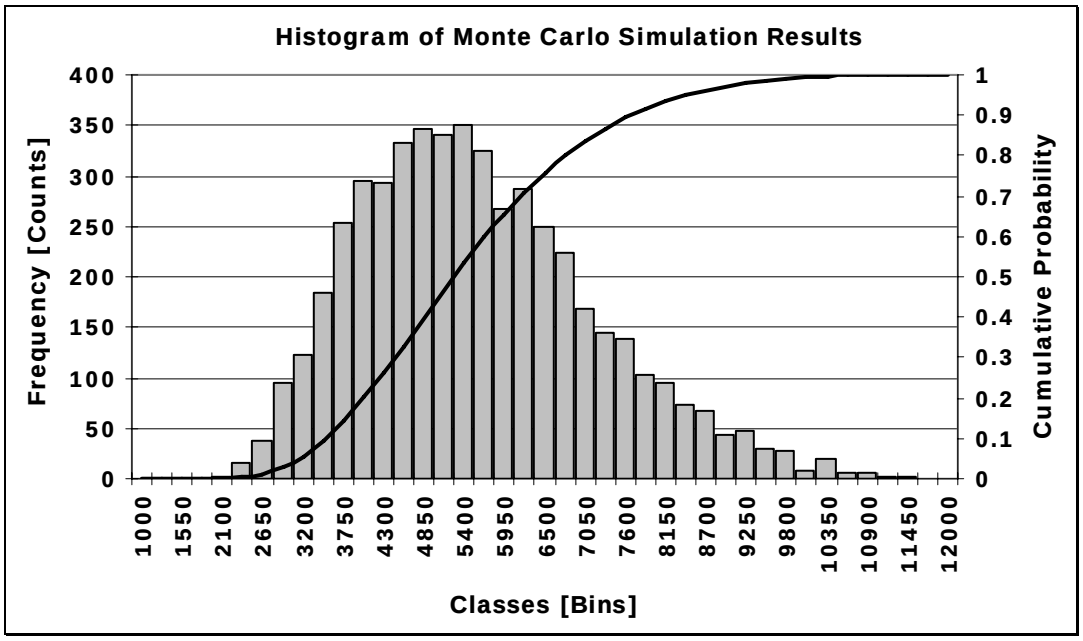
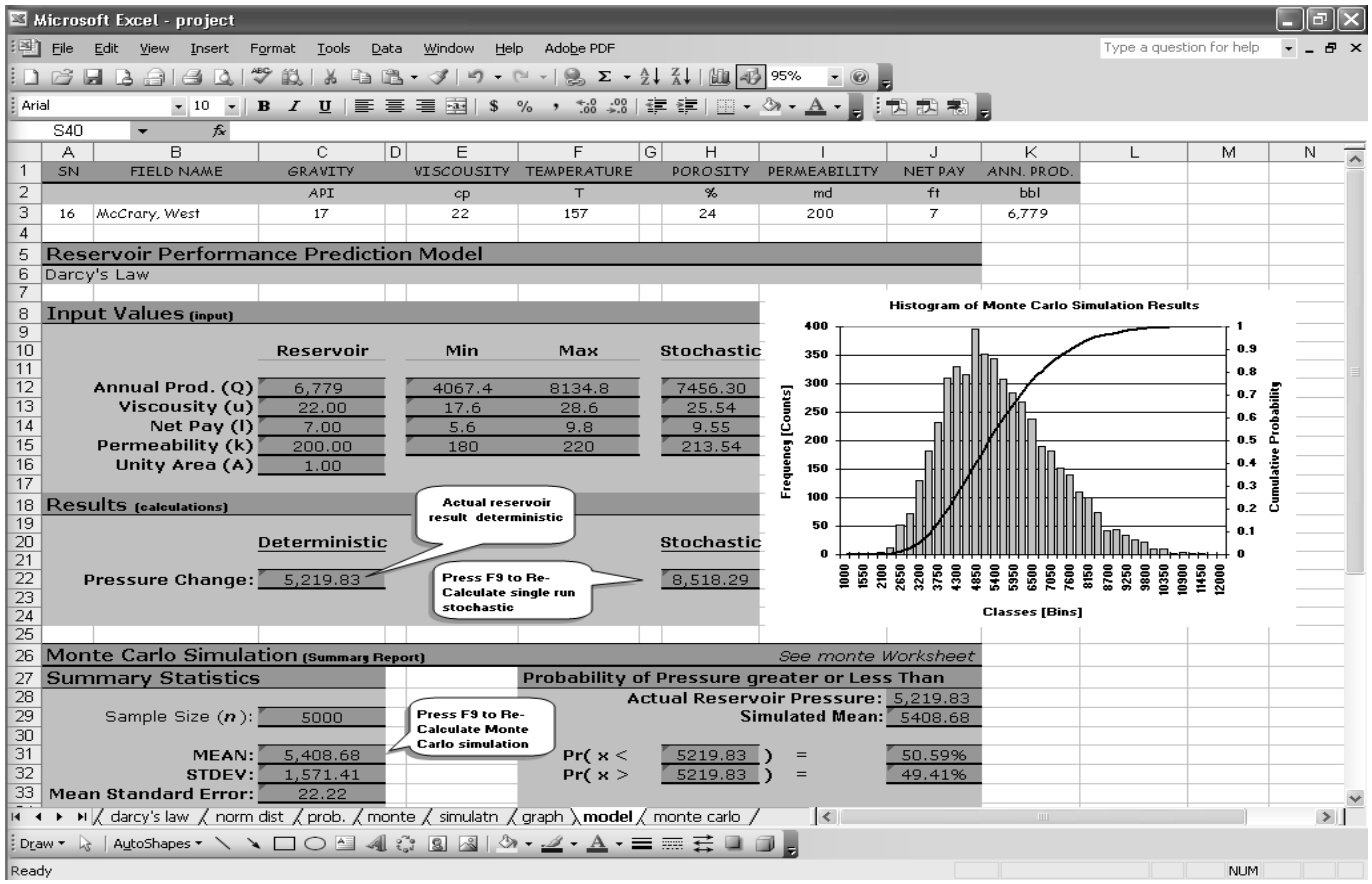


Figure 7 Histogram and probability distribution of the Monte Carlo simulation.

A sample screen of the MS Excel calculations with the histogram is shown below.



CONCLUSIONS

Computer Simulation has to do with using computer models to imitate real life or make predictions. The main ingredient of a stochastic simulation using Monte Carlo is the analysis of a particular problem using random genrated data. When a model created with a spread sheet like MS Excel, we have a certain number of imput parameters and a few equations that use those inputs to give a set of outputs (or rrespnse variables). This type of model is usually deterministic meaning we get the same results no matter how many times you recalculate as shown below.

Monte Carlo Simulation is a method for iteratively evaluating a deterministic model using sets of random numbers as inputs. This method is often used when the model is complex, non linear, or involves more than just a couple uncertain parameters. A simulation can typically involved over 10,000 evaluations of the model, the higher the number of evaluation, the more accurate the results.

Our work analysed 5000 runs using random inputs. By using random inputs, we are essentially turning the deterministic model into a stochastic model which is then solved iteratively over the chosen number of runs.

The deterministic model used in darcy equation and it was evaluated using a single well data from the sample shown earlier. A comparison of the deterministic result [5,219.83], the single run stochastic method [3,565.34] and the Monte Carlo Simulation [5430.49] shows the advantage of the multiple run achieved through the Monte Carlo Method.

The result being quite close to the eterministic value thus demonstrated that the Monte Carlo iteration can achive a high enough accuracy to be reliable in evaluating any suituation. Various statistical tools were also employed to display the standard deviation of the range of generated data from the mean and the standard error was also calculated. They are all points to the reliability of the approach.

List of Symbols

q = Production Rate	bbls/day
k = Permeability	Darcy
A = Cross section area	m^2
μ = Fluid viscosity	Kg/m.s
$\frac{\Delta P}{L}$ = Pressure gradient (Pressure change per unit length).	Pascal/m

* Corresponding author, key words, reservoir performance, stochastic, Monte Carlo, simulations

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